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Report S 202

Performance trial no VII on flame radiation. Statistical analysis of the data.

part II

by

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0. Introduction

In Report S 201 of the Statistical Department of the Mathematical Centre, the general analysis of the observations, obtained during Performance trial no VII has been discussed and results regarding the infuuences of the experimental conditions on R_1, R_2, R_3 and e have been stated.

This report deals with a number of questions of a more specific character, viz.:

1. The relation between the temperature of the flames, measured in a direct way, and the temperature, calculated from the observed radiation by means of the laws of radiation.
2. The influence of the four factors (fuel, momentum, amount of air and combustion air temperature) on the mixedness-coëfficient.
3. The influence of the temperature of the combustion air on the amount of measured carbon.
4. The relation between the amount of measured carbon and the emissivity coefficient e.
5. It was noticed on plotting the experimental results that the difference between the observations on R_1 (slot 6 and 7) of flame 1 and 2 is much bigger than between flames 3 and 4, 5 and 6, etc. Are there any indications whether this difference is significant.
6. Flames have been observed under two set of conditions (indicated by XVII and XVIII) which did not fit in with the general 2^4 factorial scheme. It is of interest to compare the experimental results of flame XVII with these of flames III, and of flame XVIII with those of the flames III and IV.

As a rule a set of indices will be attached to the symbols used for the observations of the mixedness coefficient, temperature, etc., to indicate the experimental condition under which these particular observations were made. Though these indices are the same as used in report S 201, we shall state them again for easy reference.

s = slotnumber (s = 2,...,7)

i = 1 for oil } A factor
    2 for gas

j = 1 for low momentum } B factor
    2 for high momentum

k = 1 for 110% stochiometric air } C factor
    2 for 140% " "

$l = 1$ for combustion air of 100° C
$\quad 2$ " " " " 650° C } D factor

$m = 1$ for up
$\quad 2$ for down

$n = 1$ for 1st replicate
$\quad 2$ " 2nd "

$O = 1$ for 1st observation of a replicate
$\quad 2$ " 2nd " " " "

Throughout this report we have made the assumption of normality of the distributions, of which every observation can be regarded as a sample of size one. It is not known to what extent this assumption is true, but experience has shown, that in cases where this assumption is only approximatively true, the results are fairly reliable.

Test results in this report have been expressed in terms of "tail probability", which is the size of smallest one- or two-sided critical region just containing the observed value of the test statistic. If the tail probability $\leq$ 0,05, the null hypothesis can be rejected at a level of significance of 5%, etc.

Often this will be indicated by a roman I. In case the null hypothesis can be rejected at a level of significance between 1% and 0,1% (0,1% and 0,01% respectively) we shall indicate this by II and III respectively.

1.1. The relation between the measured and the "calculated" temperature

From observations of the radiation of a flame a temperature T_f can be calculated by means of the laws of radiation. Moreover a temperature T_p has been measured directly at the same times and the same places as the radiation. It is asked to examine the relation between T_f and T_p.

As a rule one measurement of T_p and two calculated values of T_f were available for each slot and each of the values of the n-factor. Therefore the statistical analysis has been based on the mean of the said two values. The measurement of T_p and the mean of the two values of T_f respectively will be indicated by T_{fsijkl} and T_{psijkl} respectively.

In plotting the observations it appeared to be essential to perform the analysis separately for the case of oil and that of gas flames.

1.2. The relation between T_f and T_p for oil flames

We choose the following model:
Every observed value T_{ps1jkl} in an observation of a random variable $\underline{T}_{ps1jkl}$ and

$$\underline{T}_{ps1jkl} = T_{fs1jkl} \cdot \beta_{s11} + \alpha_{s11} + \underline{V}_{s1jkl} \quad \ldots \qquad (1.2;1)$$

The following assumption is made:
The random variables $\underline{V}_{s1jkl}$ have a normal distribution with mean 0 and variance σ^2_1, moreover they are all independent of each other. The model implies that the relation between T_p and T_f does not depend on j and k (momentum and stochiometric air).

For every slot and for l = 1 and l = 2 the usual regression analysis can be used to obtain estimates a_{s11} and b_{s11} of α_{s11} and β_{s11} respectively. E.g. to calculate the estimate b_{311} and a_{311}, all observations taken at the 3rd slot are used, for which i = 1 and l = 1 (flames I, III, V and VII). The observations of flames XVII (i = 1, l = 1) and XVIII (i = 1, l = 2) have been included in the regression analysis, as the connection between T_f and T_p has been assumed to be independent on j and k.

Regarding α_{s11} and β_{s11} several tests have been applied. For every value of s the hypothesis

$$H_o: \ \beta_{s11} = 0 \text{ and } \beta_{s12} = 0$$

has been tested (test A) by means of a covariance-analysis test. If this test A leads to rejection of H_o, the conclusion can be drawn that a significant regression of T_p on T_f exists. If the test does not lead to rejection, a connection between T_p and T_f cannot be proved to exist.

At this point it can be remarked that assumption 1 implies that if a significant regression exists, the relation between T_p and T_f is of linear kind. This restriction has been adopted in view of the small number of observations available and because of this the validity of this assumption cannot be tested.

In case test A leads to rejection of H_o, it is possible to examine the influence of the 1-factor (temperature of combustion air) on the relation between T_p and T_f. A test B can be applied for the null hypothesis

$$H_o': \beta_{s11} = \beta_{s22} .$$

If this test shows no reason to reject H_o', a third hypothesis can be tested (test C), viz.

$$H_o'': \beta_{s11} = \beta_{s12}$$

and

$$\alpha_{s11} = \alpha_{s12} .$$

The conclusions that can be drawn from the outcomes of these tests are stated for every possible case in Table 1.1.

In this report we have omitted the details of the actual testing procedures. They can be found in almost every handbook on statistics, e.g. DIXON and MASSEY [1] and KENDALL [2] .

It may be noticed that though test B has only been applied when test A rejected H_o , this does not influence the level of significance of test B. Actually test B could equally well be applied to the data in case test A did not lead to rejection, but then the results of the test B would not be of any interest to us.

The results of the tests are stated in Table 1.2. Table 1.2 shows that an indication of a significant T_p- T_f relation exists at the third slot (and to a minor degree at the second slot). The temperature of the combustion air seems to influence rather the level of this relation than the direction of the regression lines.

There is further an indication of a significant T_p - T_f relation towards the end of the furnace. Here the effect of the temperature of the combustion air is essentially to displace the regression line parallel to itself.

In figure 1.2.1 - 1.2.6 the observations are plotted, together with the calculated regression lines, while the regression lines of all the slots are drawn in figure 1.2.7.

Table 1.1

Testing procedures and the possible conclusion it can lead to.

			Conclusion
test A	rejects H_o - test B	rejects H_o'	A significant T_p-T_f relation exists. The temperature of the combustion air effects the slope of the regressionline.
		no rejection - test C: rejects H_o''	A significant T_p-T_f relation exists. The temperature of the combustion air changes the general level, but there is no indication that it changes the directions of the regressionline.
		no rejection - test C: no rejection	A significant T_p-T_f relation exists. The temperature of the combustion air cannot be shown to have any influence on the regressionline.
	no rejection		It cannot be proved that a significant T_p-T_f regression exists.

Table 1.2

Regression analysis of the T_p-T_f-data together with testing results (Oil flames).

slot	tail probability			
	test A	test B	test C	
2	0,05	> 0,50	> 0,50	$T_p = 330 + 0,64\ T_f$
3	< 0,001	> 0,30	0.04	$T_p = -3269+3.208T_f$ for l=1 $T_p = -3466+3.208T_f$ for l=2
4	> 0,50	-	-	-
5	>0.50	-	-	-
6	0,01	> 0,50	≪ 0,001	$T_p = 991 + 0.29T_f$ for l = 1 $T_p = 1120 + 0.29T_f$ for l = 2
7	0.05	>0.50	0,002	$T_p = 873 + 0.36T_f$ for l = 1 $T_p = 988 + 0.36T_f$ for l = 2

1.3. The T_p - T_f relationship for gas flames

Essentially the same analysis has been applied to the data obtained on gas flames.

The following model has been chosen:

Every observed value T_{ps2jk} is an observation of a random variable $\underline{T}_{ps2jk}$:

$$\underline{T}_{ps2jk} = T_{fs2jkl}\,\beta_{s2} + \alpha_{s2} + \underline{V}_{s2jkl} , \ldots \qquad (1.3.1)$$

where $\underline{V}_{s2jkl}$ are random variables with mean 0. Again we make the assumption that the random variables $\underline{V}_{s2jkl}$ are distributed normally and independently of each other. Their (common) variance is σ_2^2.

Flames XVII and XVIII supply additional information regarding the T_p-T_f relation for oil. No corresponding gas flames have been observed and therefore a smaller number of observations is available for gas than for oil. This is why the analysis of the experimental resultst cannot be as detailed for gas as it was for oil; and especially the influence of the l factor has been neglected. The model (1.3;1) implies that the relation between T_p and T_f does not depend on j, k and l.

The test A has been applied for every s to test the null hypothesis

$$H_o: \beta_{s2} = 0$$

If the test leads to rejection, it means that a significant T_p-T_f relation exists. If the test does not lead to rejection, the existence of a relation cannot be proved from the observations.

The results of the regression analysis, together with the testing results are stated in Table 1.3.

Table 1.3

Regression analysis of the T_p-T_f data together with testing results (Gas flames)

slot	tail probability test A	regression line
2	> 0.10	-
3	0.40	-
4	1.00	-
5	0.03	$T_p = 586 + 0.78T_f$
6	0.02	$T_p = 659 + 0.66T_f$
7	< 0.001	$T_p = 382 + 0.82T_f$

From the results stated in Table 1.3 it can be concluded that a significant relation between T_p and T_f exists towards the end of the furnace. It is in this part of the furnace that gas and oil flames apparently behave similar, regarding the T_p-T_f relation.

Further research can show whether the T_p-T_f relation is independent of the l-factor or whether this assumption cannot be maintained.

The observations are plotted in fig. 1.3;1-1.3;4, where the regression lines are drawn as well. The regression lines of all the slots are brought together in fig. 1.3.5.

2.1. The mixedness coefficient

Observations of the mixedness coefficients were available for every slot and every set of experimental conditions. The mixedness coefficient has sometimes been measured twice (one time for each replicate of a flame), but in a large number of cases only one observation was made.

Moreover the carneau values of the mixedness coefficients have been supplied.

A preliminary analysis of variance showed the necessity of analyzing separately the data for oil and gas.

We shall denote an observation by x_{sijkln}. This x_{sijkln} can be regarded as an observation of a random variable $\underline{x}_{sijkln}$.

The following assumptions will be made:

1. for every s and i:

$$\underline{x}_{sijkln} = \mu^{(si)}_{...} + \mu^{(si)}_{j..} + \mu^{(si)}_{.k.} + \mu^{(si)}_{..l} + \mu^{(si)}_{jk.} + \mu^{(si)}_{j.l} + \mu^{(si)}_{.kl} + \mu^{(si)}_{jkl} + \mu^{(si)}_{jkln} + \varepsilon^{(si)}_{jkln} \qquad (2.1;1)$$

with the constraint:

$$\sum_j \mu^{(si)}_{j..} = \sum_k \mu^{(si)}_{.k.} = \sum_l \mu^{(si)}_{..l} = \sum_j \mu^{(si)}_{jk.} = \sum_k \mu^{(si)}_{jk.} = \ldots = \sum_j \mu^{(si)}_{jkl} = \sum_k \mu^{(si)}_{jkl} = \sum_l \mu^{(si)}_{jkl} = 0 . \qquad (2.1;1)$$

In this mathematical model

$\mu^{(si)}_{...}$ is the general mean

$\mu^{(si)}_{j..}$ is the B main effect,

$\mu^{(si)}_{jk.}$ is the B x C interaction effect,

all for slot s and the type of fuel denoted by i.

2. The random variables $\epsilon^{(si)}_{jkln}$ are distributed normally and except from s independently of each other with mean 0 and variance σ^2_s.

The scheme of the corresponding analysis of variance is given in Table 2.1.

Table 2.1

Analysis of variance of the observations of the mixedness coefficient

source of variation	degrees of freedom	Sum of squares	Expected mean sum of squares
B	1	$8\sum_j (x_{sij...} - x_{si....})^2$	$8\sum_j (\mu^{(si)}_{j..})^2 + \sigma^2_s$
C	1	. . . .	. . . .
D	1	. . . .	. . . .
BC	1	$4\sum_{j,k}(x_{sijk..} - x_{sij...} - x_{si.k..} + x_{si....})^2$	$4\sum_{j,k}(\mu^{(si)}_{jk.})^2 + \sigma^2_s$
BD	1	. . . .	. . . .
CD	1	. . . .	. . . .
BCD	1	$2\sum_{j,k,l}(x_{sijkl.} - x_{sijk..} - x_{sij.l.} + x_{si.kl.} + x_{sij..} + x_{si.k.} + x_{si..l} - x_{si...})^2$	$2\sum_{j,k,l}(\mu^{(si)}_{jkl})^2 + \sigma^2_s$
Remainder	8	$\sum_{j,k,l,n}(x_{sijkln} - x_{sijkl.})^2$	σ^2_s

As in a number of cases only one observation for a set of experimentel condition had been made, a simple missing plot technique has been used to adjust for this. E.g. at slot 2 for i = 1, j = 1, k = 1 and l = 1 only one observation, viz. for n = 1, is available, but for the other combinations of experimental conditions observations are available. The calculation of the mean sum of squares proceeds as if x_{21112} has the same numerical value as x_{21111}, while the number of degrees of freedom for the remainder in Table 2.1 is lowered by one. If in this case the mean sums of squares of the effects and the interactions are multiplied by $\frac{16}{16+1}$, the F test remains approximately valid. The generalisation of this procedure for the case when p values are missing, but where for every set of values i, j, k and l at least one observation is available is quite straight forward (the multiplication factor is then $\frac{16}{16+p}$).

2.2. The mixedness coefficient for oil flames

In table 2.2 the results of the analysis are given. The figures give the estimated effects of the low level of the independent variable (low momentum, 110% stochiometric air, 100° C), as far as the main effects are concerned. If we denote the low levels of the factors with a + sign and the high levels with a - sign, all interactions get also allocated a + or - sign by multiplying the signs of the factors involved. The effects tabulated are those which have a + sign attached to it.

E.g. flame 5 of slot 4: significant contributions to the expected value are the effects: B, BC and the general mean. B occurs at its higher level, and therefore contributes -(-0.038) = + 0.038. The BC interactions effect is + 0.028, and as for flame 5 B occurs at its higher and C at its lower level, this has to be multiplied by -1. Finally we find for the expected value of the mixedness coefficient + 0.825 - 0.028 + + 0.038 = 0.825. The actual observed values were 0.830 and 0.830.

The roman figures in Table 2.2 denote the level of significance, as has been explained in the introduction.

The results stated in Table 2.2 can be summarized as follows:

The momentum can be seen to have a fairly large influence throughout the furnace, though this effect does not attain significance at slot 7 and is definitely zero for the carneau (however it should be born in mind that for slot 7 the remainder term of the analysis of variance has only two degrees of freedom and because of this, the power of the test is very low). The influence of the momentum is not independent of the amount of air (the large BC interaction). The C main effect (amount of air) is highly significant towards the end of the furnace. The size of the effect can be seen from graph 2.2;1.

The temperature of the combustion air (D) has a definite effect at the slots 5 and 6. Though not being significantly different from zero, the estimate of the effect at slot 7 still shows a fairly high value, but is very small beyond this slot (See fig. 2.2;1 and 2.2;2).

Table 2.2

Estimates of the effects of the experimental conditions on the mixedness coefficient (oil flames only)

slot / effect	2	3	4	5	6	7	carneau
B	-0.058 I	-0.055 II	-0.038 (I)	-0.031 II	-0.028 II	+0.012	0.000
C	+0.012	-0.007	-0.025	-0.062 II	-0.116 III	-0.165 III	-0.152 III
D	+0.013	+0.007	+0.017	+0.024 I	+0.051 III	+0.025	+0.003
BC	+0.012	+0.023	+0.028 II	+0.051	+0.040	+0.010	+0.011
BD	+0.002	-0.007	-0.009	+0.016	-0.010	+0.011	-0.022 I
CD	-0.006	+0.019	-0.003	-0.009	-0.018	-0.018	+0.006
BCD	-0.035	-0.015	+0.003	+0.006	-0.002	-0.009	+0.008
general mean	0.435	0.640	0.825	1.023	1.173	1.243	1.240

2.3. The mixedness coefficient of gas flames

Table 2.3 shows the estimates of the effects and the results of the corresponding analysis of variance tests (see also fig. 2.3;1 and 2.3;2).

The first order interactions are rather large (BC and BD) and this makes a once and for all interpretation of the main effects rather difficult. The graphs 2.3;1 and 2.3;2 shows estimates for the effects.

For slot 7 the remainder term of the analysis of variance carried only one degree of freedom, and the power of the tests based on this remainder term is very small; even the physically well established C main effect cannot be proved to be significantly different from 0.

Table 2.3

Estimates of the effect of the experimental conditions on the mixedness coefficient (gas flames)

effect \ slot	2	3	4	5	6	7	carneau
B	-0.018	-0.025	-0.029 II	-0.012	+0.011	-0.002	+0.004
C	-0.015	-0.005	-0.041 III	-0.075 I	-0.118 III	-0.124	-0.158 III
D	+0.008	+0.030	-0.009	+0.035	+0.026 I	+0.034	+0.009
BC	-0.010	-0.011	+0.008	+0.029	+0.023 (I)	-0.011	-0.010
BD	+0.010	+0.014	+0.019 I	+0.021	+0.026 I	+0.002	-0.002
CD	-0.008	+0.003	+0.014	-0.001	-0.014	+0.015	-0.015
BCD	-0.007	-0.009	-0.016	+0.009	+0.004	-0.003	+0.004
general mean	0.486	0.716	0.894	1.073	1.196	1.262	1.230

The meaning of the signs of the estimates has been explained in 2.2.

Table 3.1

Effects of the experimental conditions on the amount of carbon in oil flames

effect \ slot	3	4	5
B	+0.645	+0.403 III	+0.078 I
C	+0.161	+0.351 III	+0.191 II
D	+0.032	-0.069	-0.043
BC	+0.143	+0.021	-0.012
BD	-0.431	-0.144	-0.018
CD	-0.487	-0.098	-0.045
BCD	+0.271	-0.088	-0.021
general mean	3.281	1.043	0.341

3. The amount of carbon in a flame

A number of measurements on the amount of carbon in the oil flames has been made. This number per slot and per replicate (n-factor) of each set of experimental conditions is generally one, but in quite a few cases two, and in other cases no measurements have been made. Because of this, it is rather difficult to base a correct statistical analysis on all the observations. Nevertheless it is possible to obtain information regarding the influence of the B, C and D factors on the amount of carbon for the slots 3, 4 and 5.

As only very few observations are available for o = 2, we shall omit these altogether.

We shall denote a carbon measurement by c_{sijkln}, which can be regarded as an observation of a random variable $\underline{c}_{sijkln}$.

The mathematical model used to analyse the carbon measurements is essentially the same as has been described in 2.1, if we substitute i = 1.

The estimate of the effects and the results of the analysis of variance tests are given in Table 3.1. The meaning of the signs of the estimates is the same as in Tables 2.2 and 2.3, and has been explained in some detail in 2.2.

Only the B and C factor abtain significance at slots 4 and 5. There is no indication of an effect of the D factor.

See figures 3.1;1 - 3.1;3.

4. The relation between the amount of carbon and emissivity

Though as has been pointed out in 3.1, the number of carbon measurements varies a good deal, an observed value of the emissivity coefficient $e_{sijklno}$ is known, corresponding to each carbon measurement c_{sijkln}.

It was asked to examine whether any relation exists between $\underline{c}_{sijkln}$ and $\underline{e}_{sijklno}$, and whether this relation is independent of the slot number.

In plotting all the pairs of observations c_{sijkln} and $e_{sijklno}$ it does not appear to be likely that one relationship exists, independently of the slot number. This can also be seen by the fact that the average carbon content decreases with the slot number, while the emissivity increases on the average during the first part of the furnace and then decreases.

The relationship between c and e can be examined for every slot separately, but then one meets a substantial difficulty in interpreting the results. Both c and e depend on the momentum (and possibly on other factors), and therefore it is very easy to come across spurious correlations, which have no physical meaning. In fact, the design of the experiment does not permit any definite conclusions to be drawn with regard to the relation between the carbon and the emissivity. The only way to reach a conclusion about this point is to vary the amount of carbon, while keeping all the other experimental factors at the same level.

5. Comparison of R_1 of flames no 1 and 2

In Document No F 31/a/11 a diagram of the R_1 plotted against the slot number showed a larger difference between the R_1 value of flame 1 and that of flame 2, than between other corresponding sets of flames.

As flame 2 was the first flame to be examined in the course of the experiments the possibility had to be considered that in this first experiment something went out of control. This is however very unlikely as the second observation of flame 2, which followed after quite a few other flames had been observed, shows observations of the radiation not differing significantly of those of the first replicate.

It may be noted that in the above mentioned document the difference at slot 7 between R_1 of flame 1 and R_1 of flame 2 has been drawn larger than it should be. This probably arises from the fact that 4 observations on flame 2, first replicate, and 8 observations on flame 2, second replicate have been made, against 4 and 4 respectively on flame 1. Because of this the general mean is weighted too heavily by the second replicate of the second flame. The value 6.00 in the diagram should be replaced by 5.36.

Still the difference between flame 1 and flame 2 is apparently larger than between other comparable sets. Though it should not be difficult to apply a test, the reliability of the result of this test is questionable, as the test would be set up after the observational results have been examined and the largest difference selected therefrom. One has to take into account that in an experiment of the size PT 7, some oddity is always bound to arise. The easiest and strictly the only way to examine whether an observed oddity represent a (unpredicted) real phenomenon is to repeat the experiment on that point.

6. Flames XVII and XVIII

Flames have been observed under two sets of experimental conditions, indicated by XVII and XVIII, which dit not fit in with the general 2^4 factorial scheme.

It was of interest to compare the R_1 (maximum and integrated values), R_2, R_3 and e-values of the following sets of flames:

$$\begin{array}{l} \text{XVII and III} \\ \text{XVIII and III} \\ \text{XVIII and IV} \end{array} \quad (6.1)$$

and

The amount of oil of flames XVII and XVIII is different from that of flames I-VIII, and cannot therefore be indicated by $i = 1$. Instead we shall indicate the amount of oil of flames XVII and XVIII by $i = i_1$ and $i = i_2$ respectively. Table 6.1 gives a review of the experimental conditions of the relevant flames.

Table 6.1
Experimental conditions of flames no III, IV, XVII and XVIII

factor / flame	i	j	k	l
III	1	1	2	1
IV	1	1	2	2
XVII	i_1	1	2	1
XVIII	i_2	1	2	2

The procedure for comparing the experimental results is the same for each of the sets of flames (6.1). Therefore details of the analysis will be given only of the comparison of flames XVII and III.

The procedure of comparing R_1 and R_2 is the same; also this is true for comparing R_3 and e values. Therefore only details of the comparison of R_1 and R_3 values will be given.

6.1. Comparison of R_1 value of flames XVII and III

We follow the mathematical model and the subsequent assumptions, stated in chapter 3, report S 201. We indicate a R_1 measurement by $x_{sijklno}$, being an observation of the random variable $\underline{x}_{sijklno}$.

For each slot we calculate

$$x_{si_1121...} - x_{s1121...}, \qquad (6.1;1)$$

which is the difference between the mean of the R_1-observations on flame XVII and III.

In order to test the null hypothesis

$$H_o: \qquad \underline{x}_{si_1121...} = \underline{x}_{s1121...}, \qquad (6.1;2)$$

an unbiassed estimate of the variance of $(\underline{x}_{si_1121...} - \underline{x}_{s1121...})$, being $2\sigma^2 + 4\sigma_w^2 + 8\sigma_b^2$, is required. This estimate can be obtained from Report S 201, table 3.1, 3rd line from bottom. However we shall use a slightly different estimate of $2\sigma^2 + 4\sigma_w^2 + 8\sigma_b^2$, viz. based on the variation between oil flames. This estimate (with 8 degrees of freedom) is:

$$\frac{1}{8}S_b^{*} = \frac{1}{8}\sum_{j,k,l}(x_{1jkl.n.} - x_{1jkl...})^2, \qquad (6.1;3)$$

and has been taken from the analysis, described in report S 201, chapter 6.

Under H_o

$$\underline{F} = \frac{(\underline{x}_{si_1121...} - \underline{x}_{s1121...})^2}{\frac{1}{8}S_b^{*}}$$

has a F distribution with 1 and 8 degrees of freedom respectively. Large values of F lead to rejection of H_o.

6.2. Comparison of R_3 values of flames XVII and III

For each slot we calculate the quantities

$$x_{si_1121...} - x_{s1121...} \qquad (6.2;1)$$

The variance of (6.2.1) is $2\sigma_w^2 + 4\sigma_1^2$, of which an unbiassed estimate (with 8 degrees of freedom) can be obtained from the general analysis described in Report S 201, chapter 6, as

$$\frac{1}{8}S_b^{**} = \frac{1}{8}\sum_{j,k,l}(x_{1jkln.} - x_{1jkl..})^2. \qquad (6.2;2)$$

The hypotheses H_o:

$$\underline{x}_{si_1 121..} = \underline{x}_{s1121..}$$

can now be tested by the quantity

$$F = \frac{(x_{si_1 121..} - x_{s1121..})^2}{\frac{1}{8} S_b^{**}}, \qquad (6.2;3)$$

which under H_o has a F distribution with 1 and 8 degrees of freedom. Large values of F lead to rejection of H_o.

6.3. Results

The table 6.2 - 6.4 show the estimates 6.1;1 (6.2;1), together with the testing results, indicated in the usual way by roman figures.

The results have also been plotted in the figures 6.2 - 6.4.

Table 6.2

Differences between means of observations on flame XVII and III*)

slot	R_1 max	R_1 integr.	R_2	R_3	e
2	+0.13	+0.34	+0.06	0.50	0.029
3	-0.22	+0.14	-0.54	0.60	0.060
4	+2.22 (I)	+2.11 I	+1.16	0.60	0.166 (I)
5	+3.58 II	+1.53 I	+2.15 II	0.89	0.259 I
6	+2.05 I	-0.80	+1.36	0.78	0.163 II
7	+0.60	+0.18	+1.04	0.73	0.020

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*) A + sign means that flame XVII has a higher value of R_1 (R_2, R_3, e) than flame III.

Table 6.3

Differences between means of observations on flame XVIII and III *).

slot	R_1 max	R_1 integr.	R_2	R_3	e
2	+2.19 II	+2.25 II	+2.74 III	+2.04 III	-0.004
3	+0.16	-1.71	+1.83 II	+1.99 III	-0.169 I
4	-1.34	-1.39	+0.86	+1.76 I	-0.193 I
5	-1.09	-1.34	+1.08	+1.94 II	-0.139
6	+0.02	-1.58 I	+1.26	+1.76 II	-0.006
7	+0.48	+0.39	+1.84 II	+1.76 III	-0.029

Table 6.4

Differences between means of observations on flames XVIII and IV *

slot	R_1 max	R_1 integr.	R_2	R_3	e
2	-0.70	-0.85	-0.86 I	-0.77	-0.003
3	-2.48 I	-2.33 I	-1.14 I	-0.84	-0.171 I
4	-3.31 II	-2.04 I	-2.32 I	-1.27 I	-0.156
5	-3.03 I	-1.91 I	-2.12 II	-1.26 I	-0.169
6	-0.78	-0.36	-1.29	-1.19 I	-0.031
7	-0.45	-0.39	-0.82	-1.15 (I)	-0.041

7. References

[1] W.J. DIXON and F.J. MASSEY, Introduction to Statistical analysis, Mc Graw Hill, New York, 1951.

[2] M.G. KENDALL, The advanced theory of Statistics. Griffin & Co, 1948.

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*) A + sign means that flame XVIII has a higher value of R_1 (R_2, R_3, e) than flames III or IV.

Relation between T_p and T_f

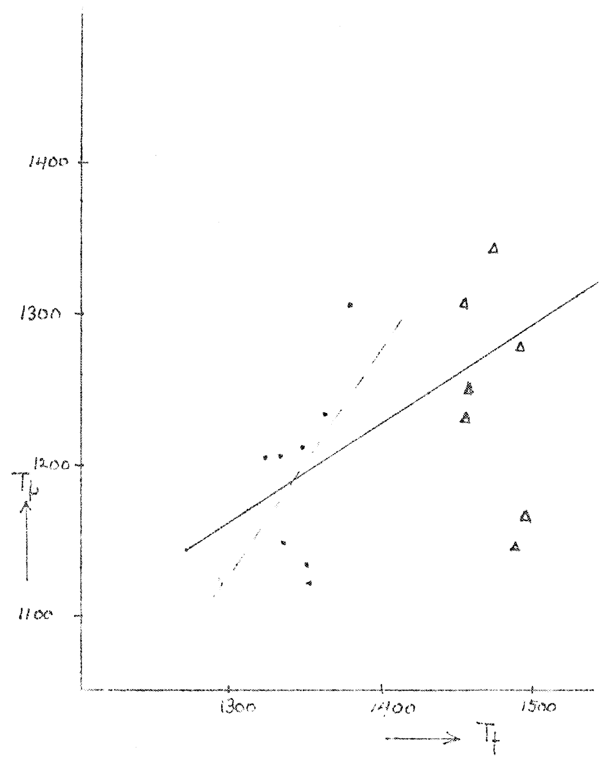


fig. 1.2;1 slot 2 (oil)

——— line based on data of 100°C and 650°C(l=1 and 2)

- - - - line based on data of 100°C only (l=1)

• observation with l=1(100°C)

Δ observation with l=2(650°C)

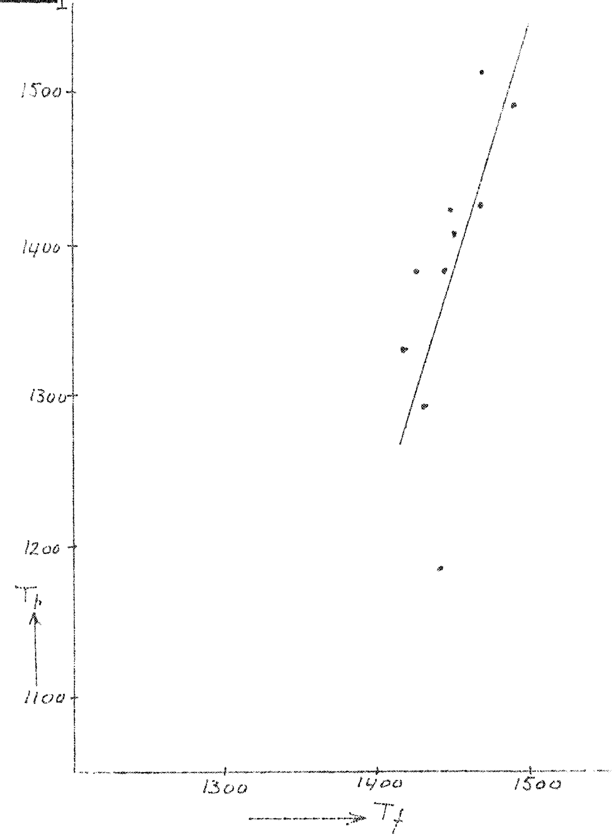


fig. 1.2;2 slot 3 (oil, l=1)

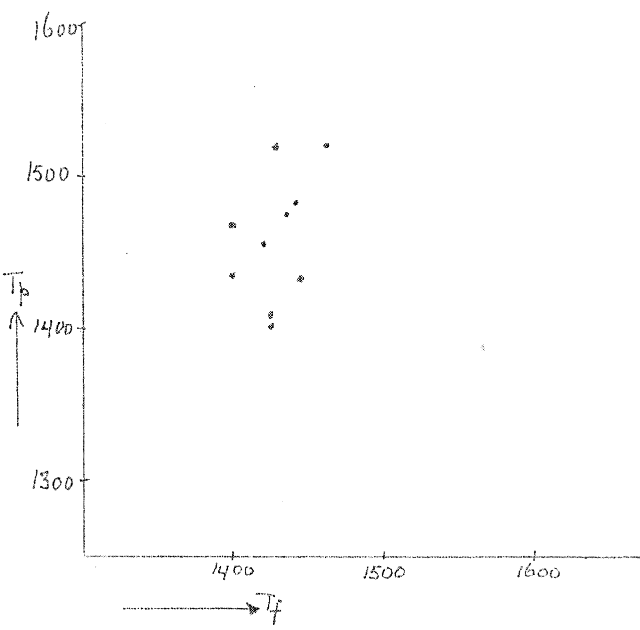


fig. 1.2;3 slot 4 (oil, l=1)

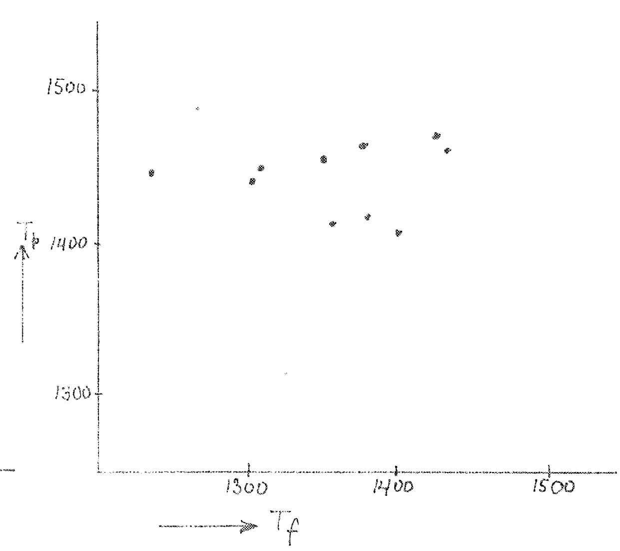


fig. 1.2;4 slot 5 (oil, l=1)

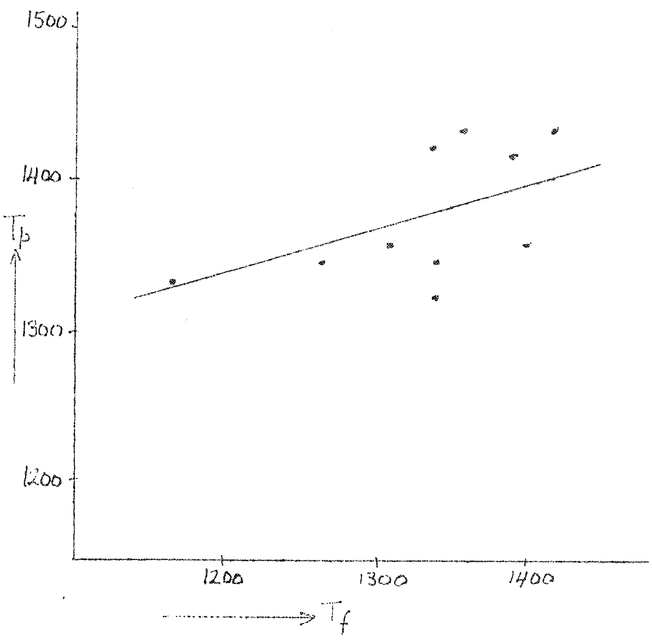


fig. 1.2;5 slot 6 (oil, l=1)

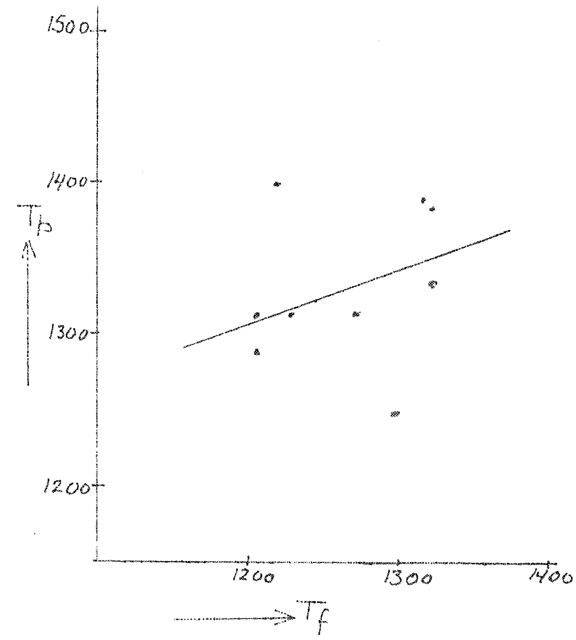


fig. 1.2;6 slot 7 (oil, l=1)

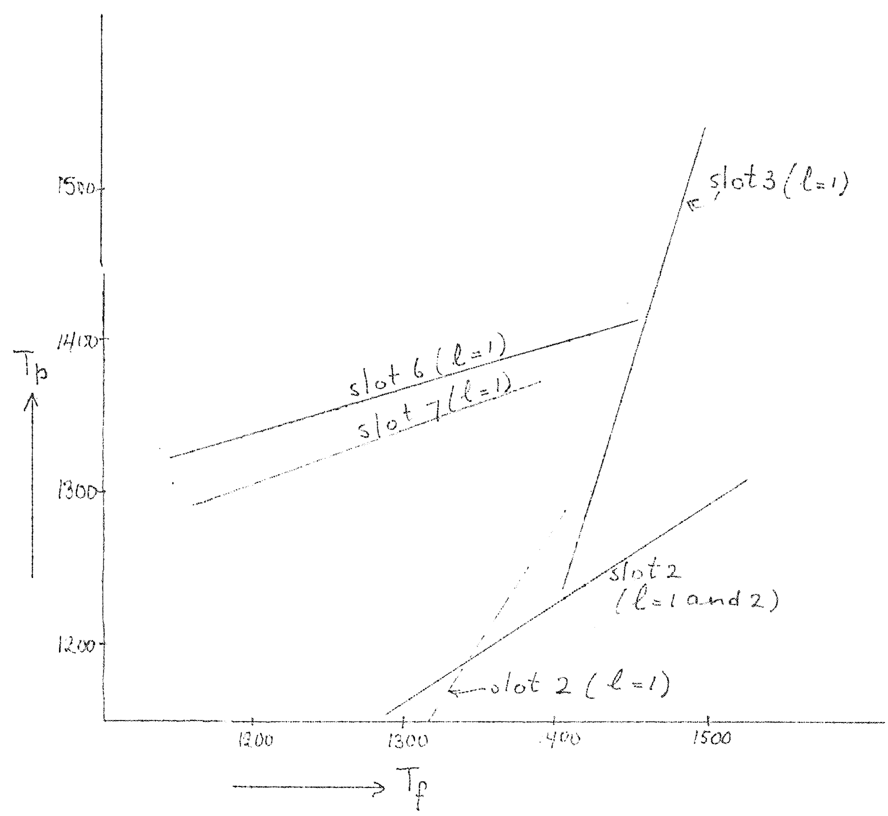


fig. 1.2;7 : The regression lines of T_p given T_f for oil flames

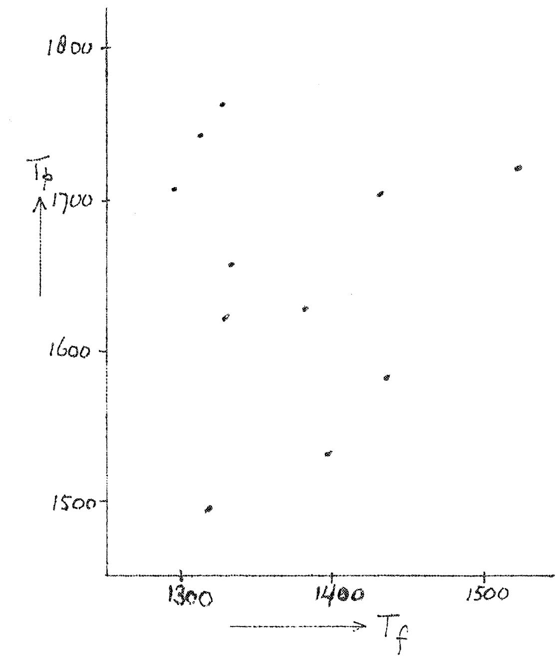


fig. 1.3;1 slot 4 (gas, l=1)

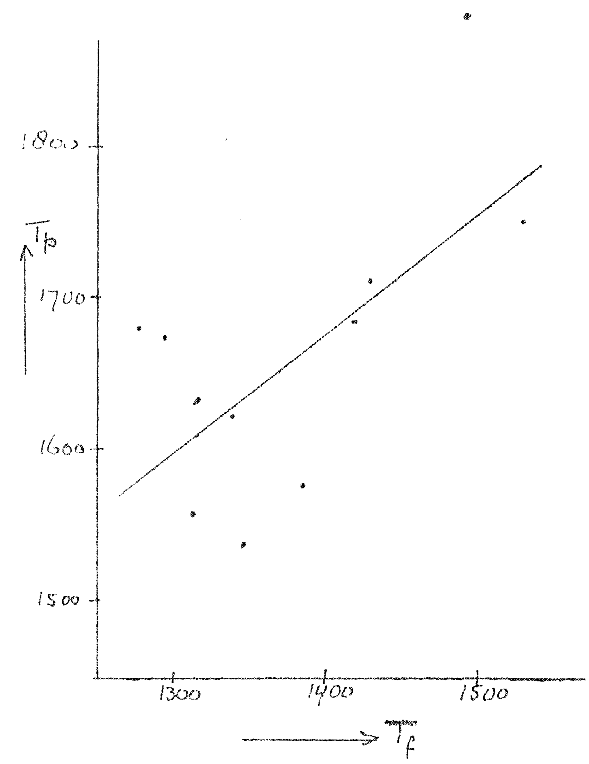


fig. 1.3;2 slot 5 (gas, l=1)

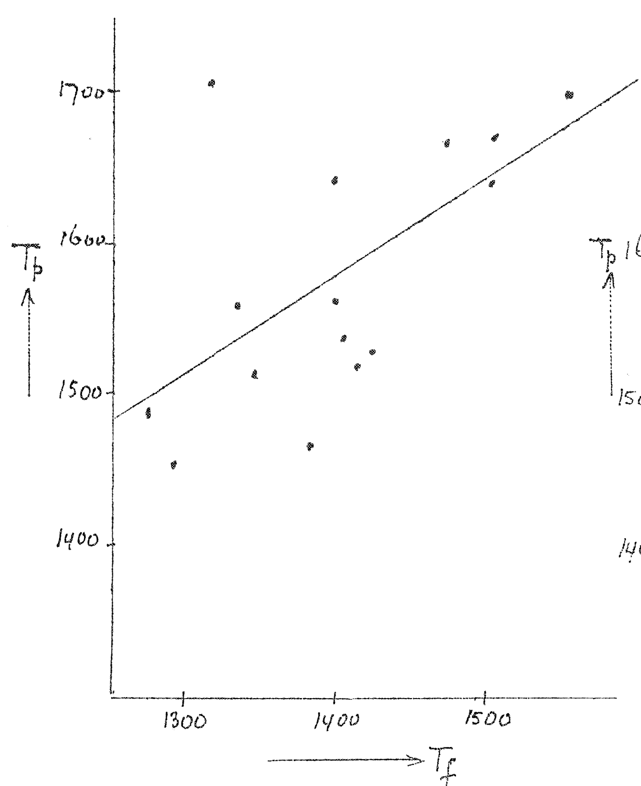


fig. 1.3;3 slot (gas, l=1)

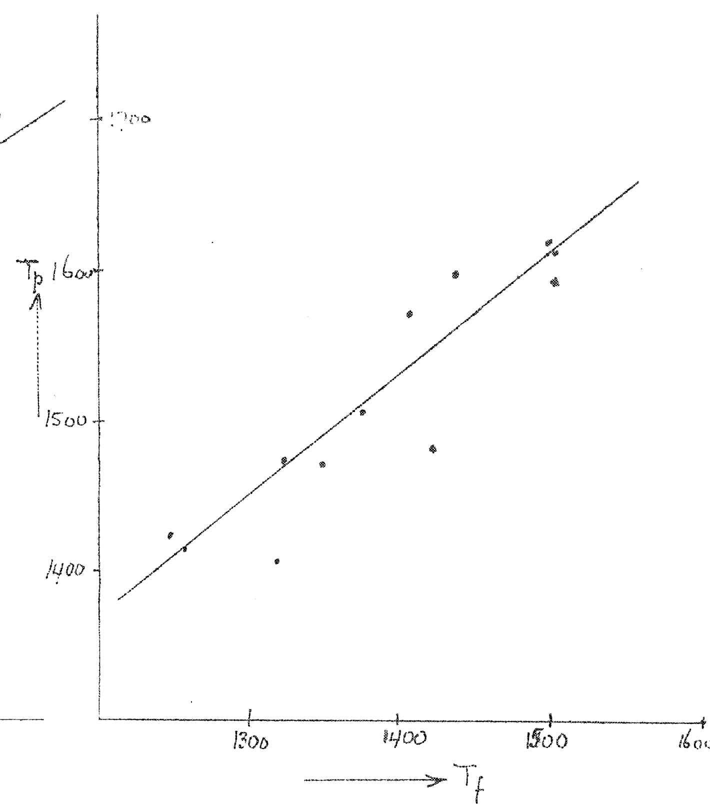


fig. 1.3;4 slot 7 (gas, l=1)

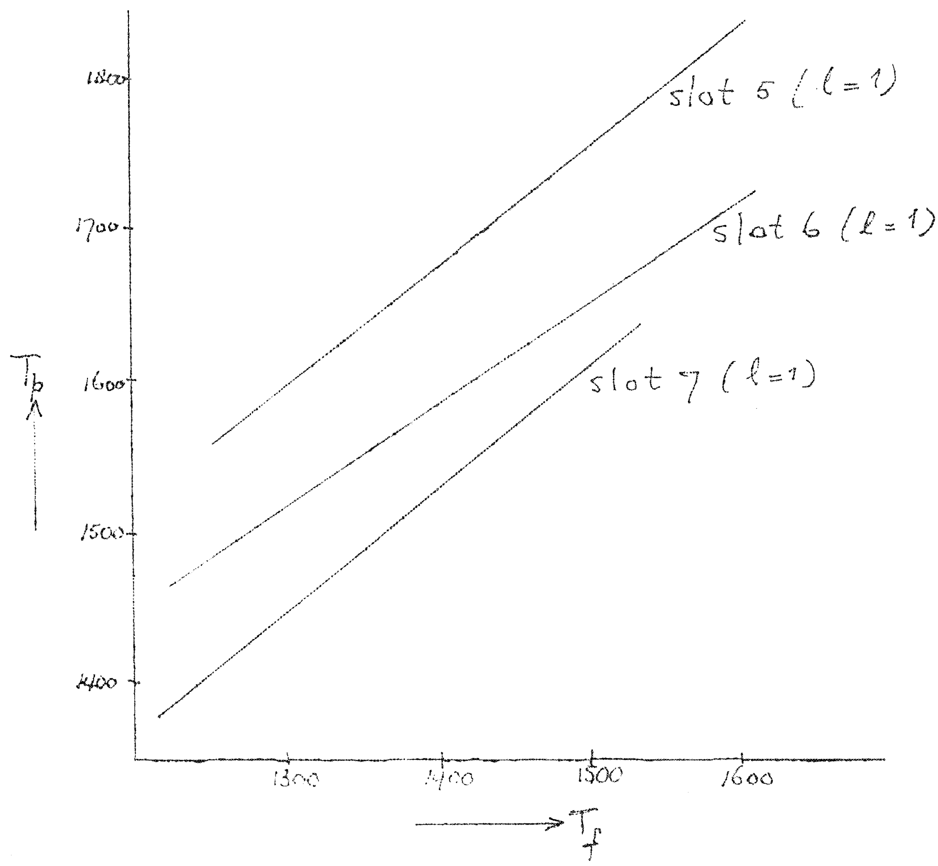


fig. 1.3;5 : Regression lines of T_p given T_f for gas flames and $100^{\circ}C$ air temperature.

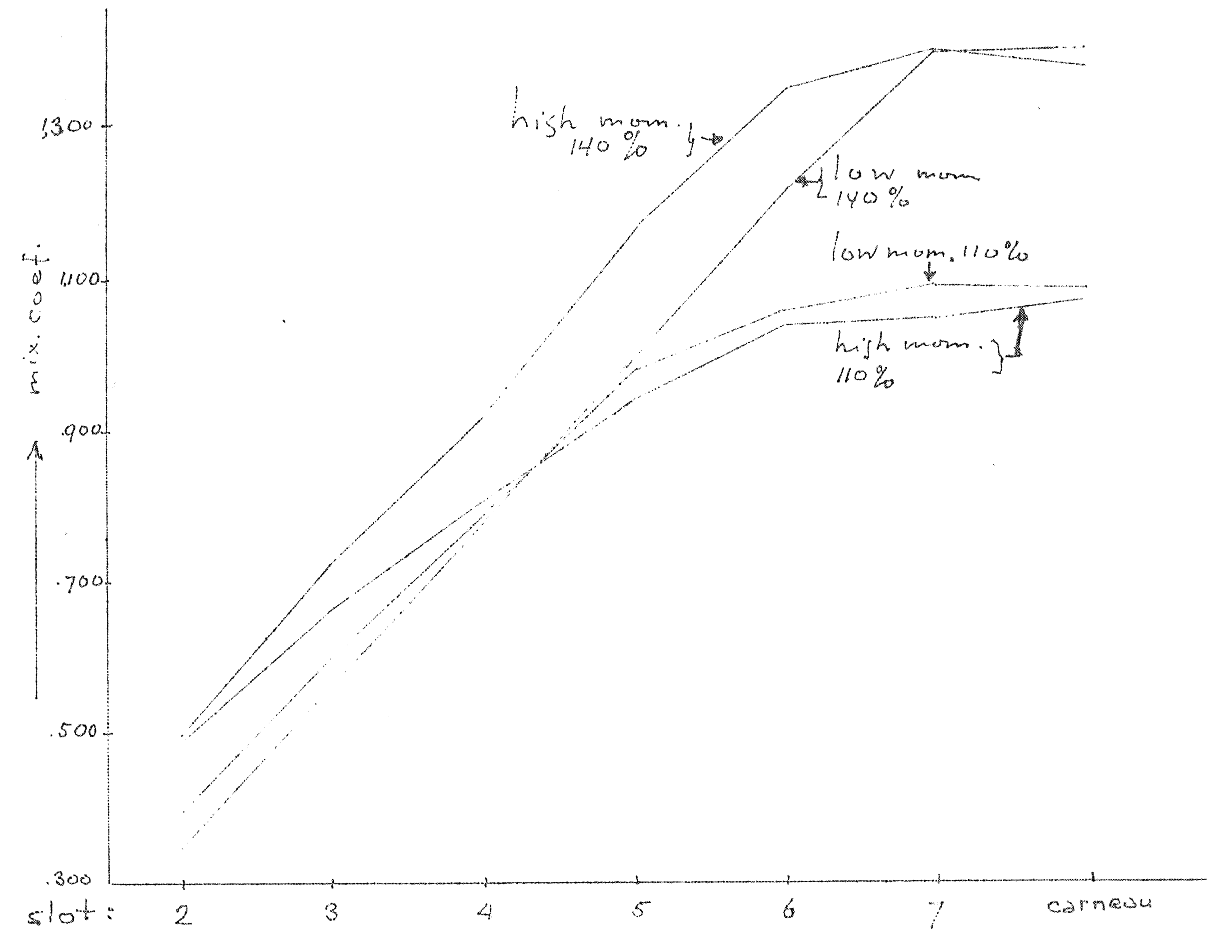


fig. 2.2;1 : <u>mixedness coefficient of oil flames</u>
influence of momentum and stochiometric air
(testing results not indicated).

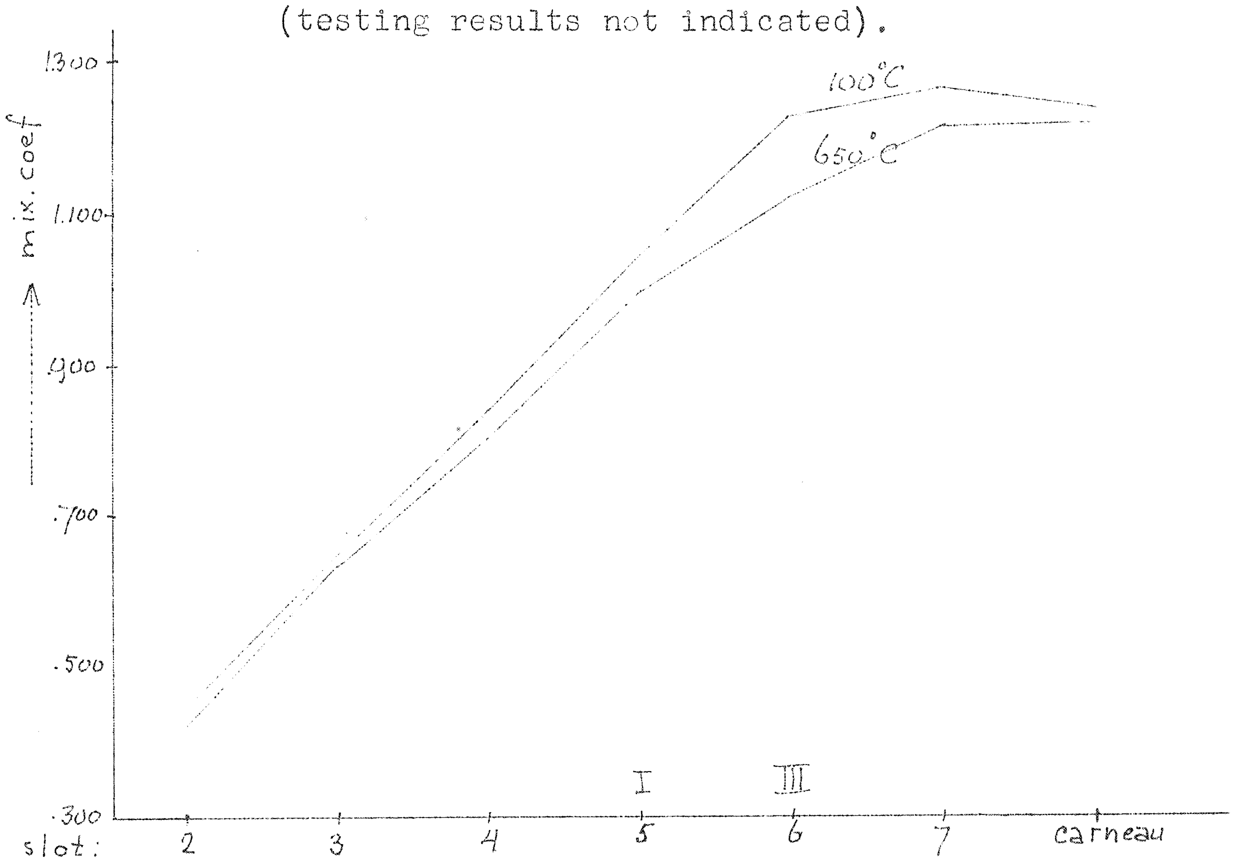


fig. 2.2;2 : <u>mixedness coefficient of oil flames</u>
influence of temperature of combustion air.

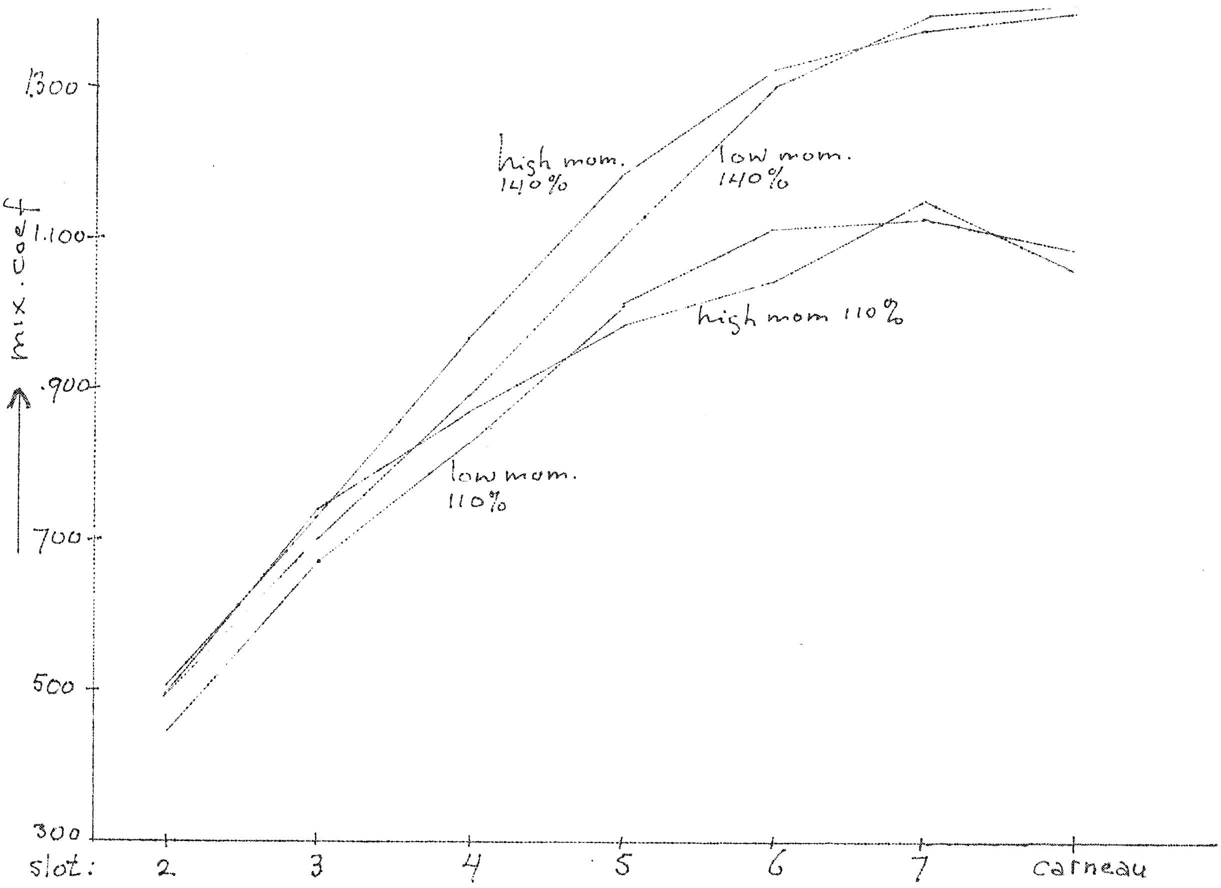


fig. 2.3;1 : <u>mixedness coefficient of gas flames</u>
influence of momentum and stochiometric air.
(testing results not indicated)

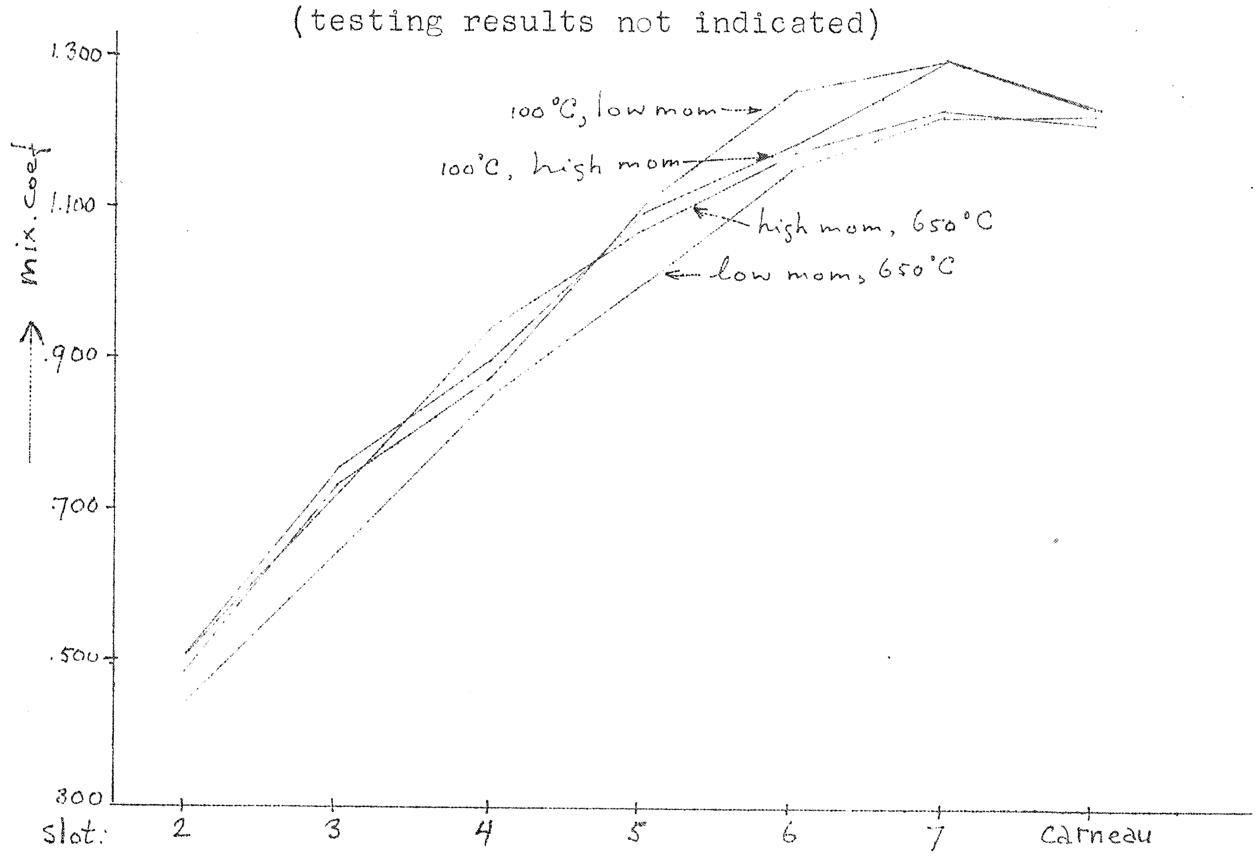


fig. 2.3;2 : <u>mixedness coefficient of gas flames</u>
influence of momentum and temperature of combustion air (testing results not indicated).

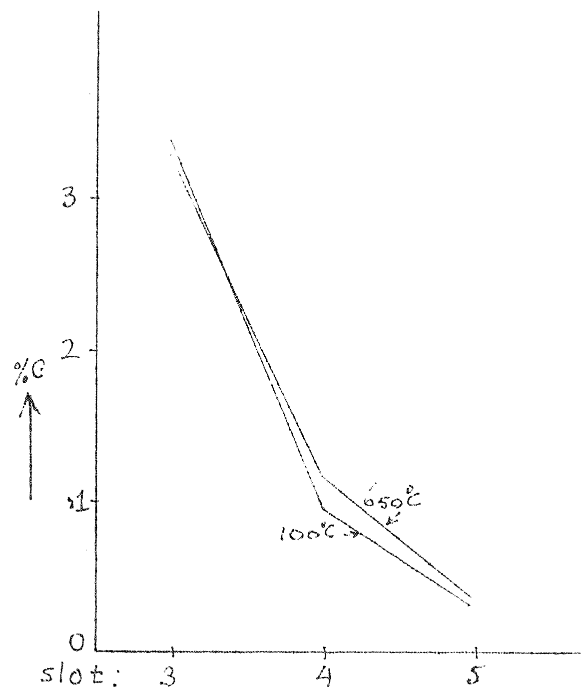


fig. 3.1;1 : influence of the temperature of the combustion air ån the carbon.

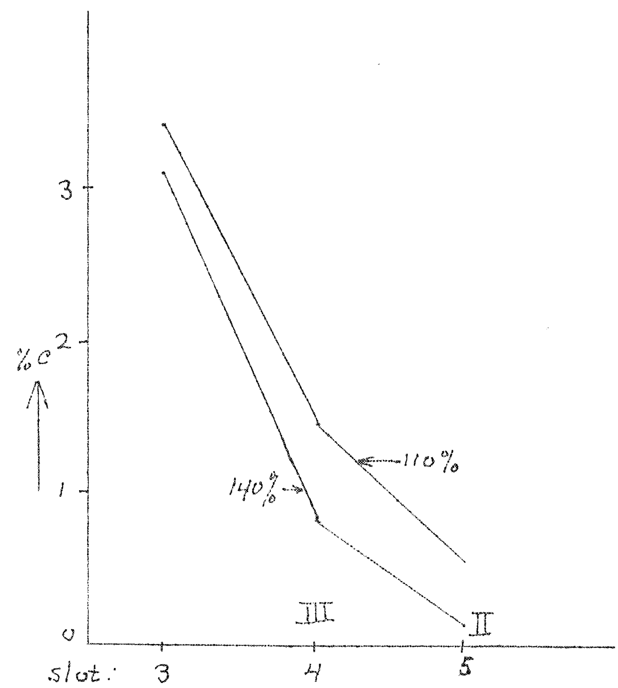


fig. 3.2;1 : influence of the stochiometric air on the carbon.

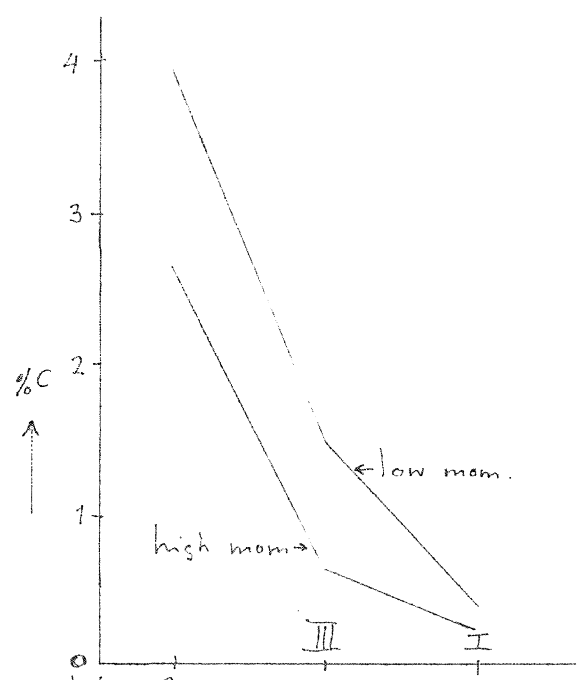
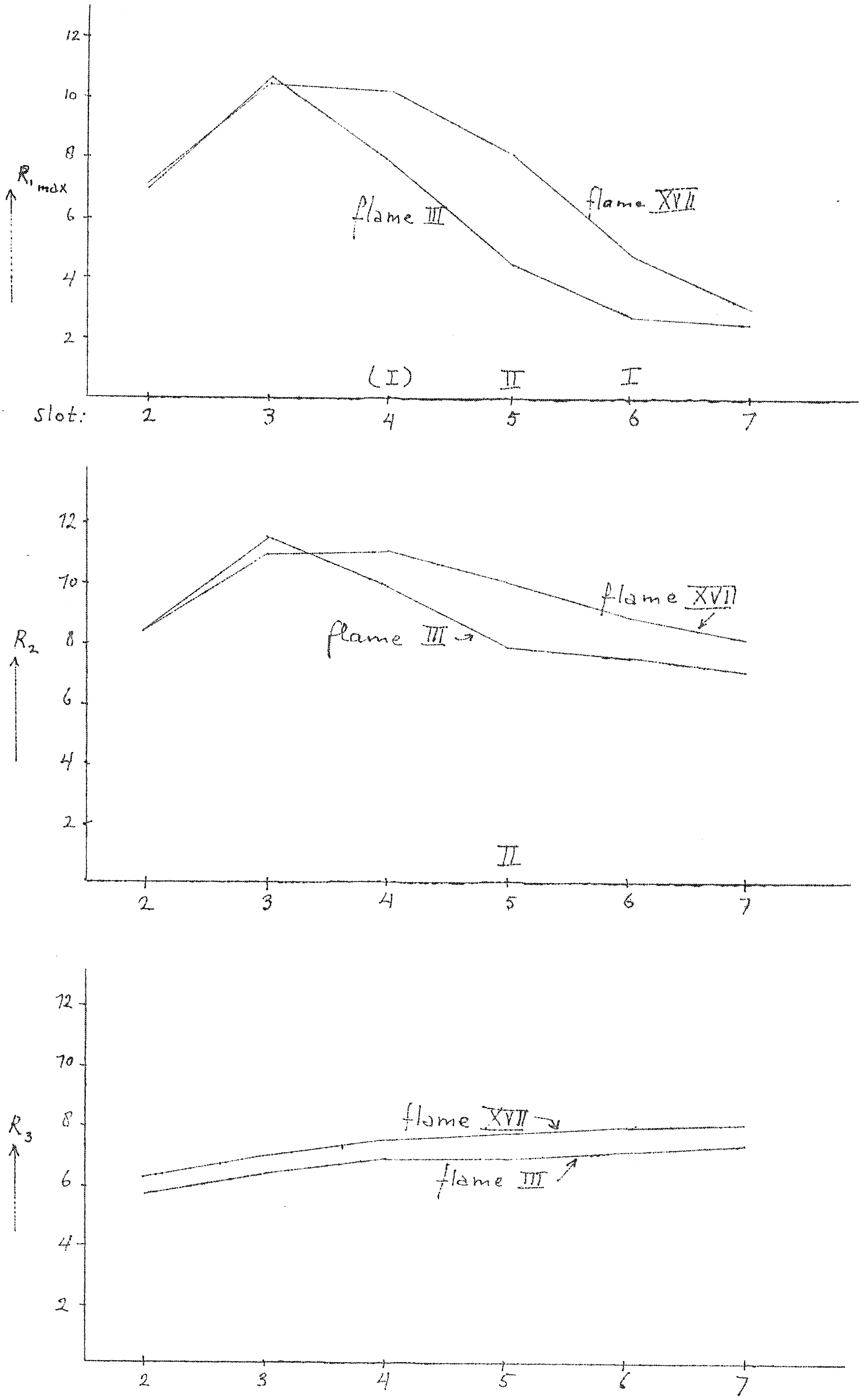


fig. 3.3;3 : influence of the momentum on the carbon.

Fig. 6.2: Comparison of the flame XVII and III:



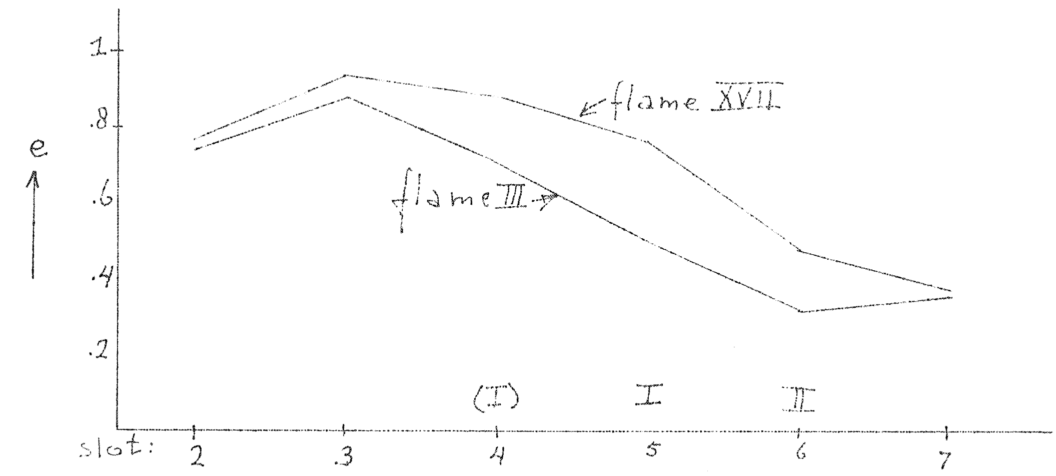
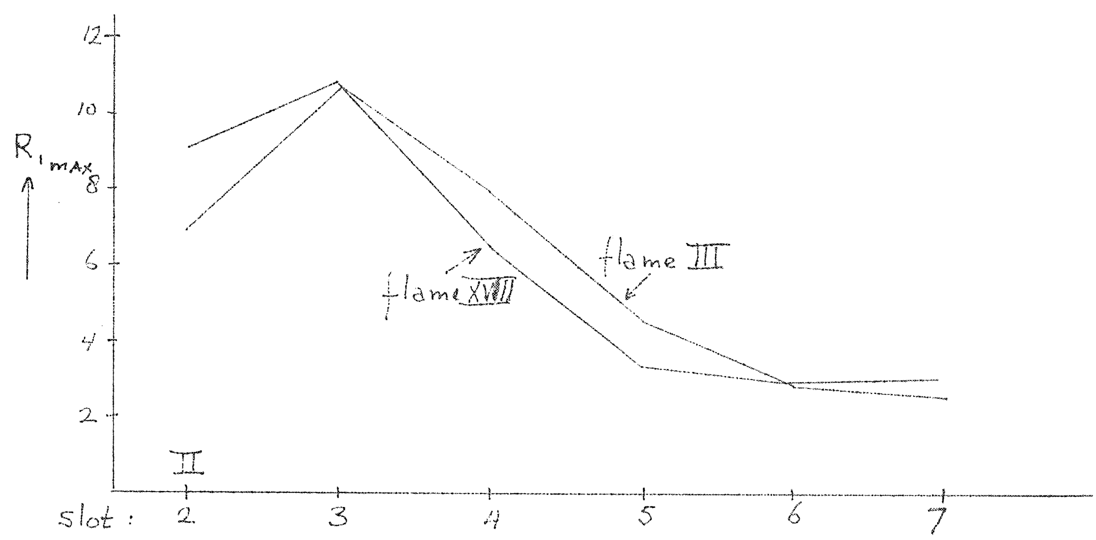
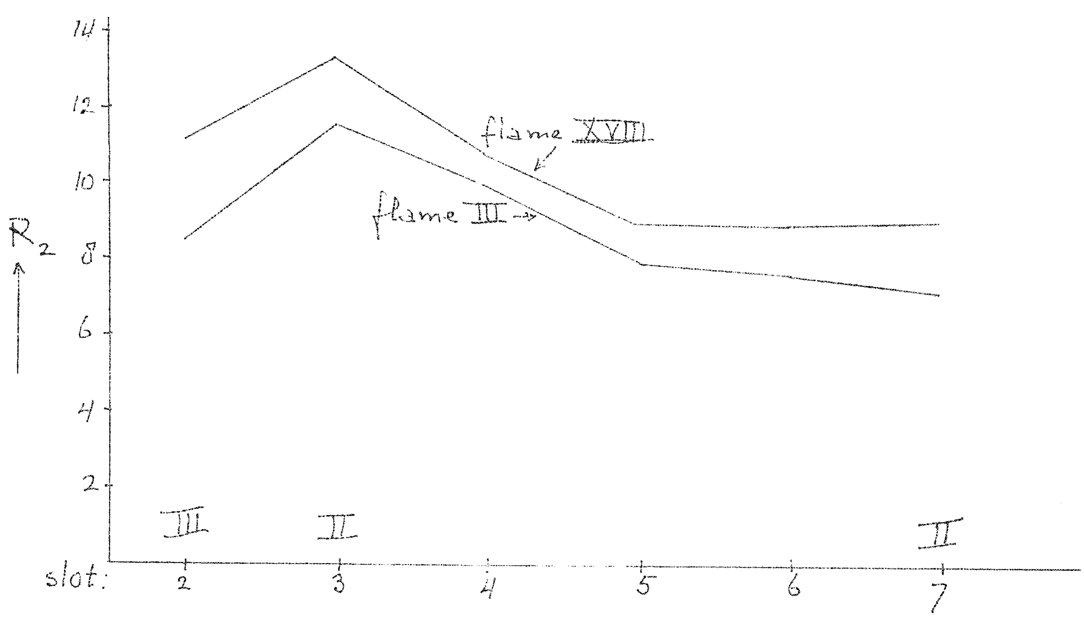


Fig. 6.3: <u>Comparison of the flames XVIII and III:</u>





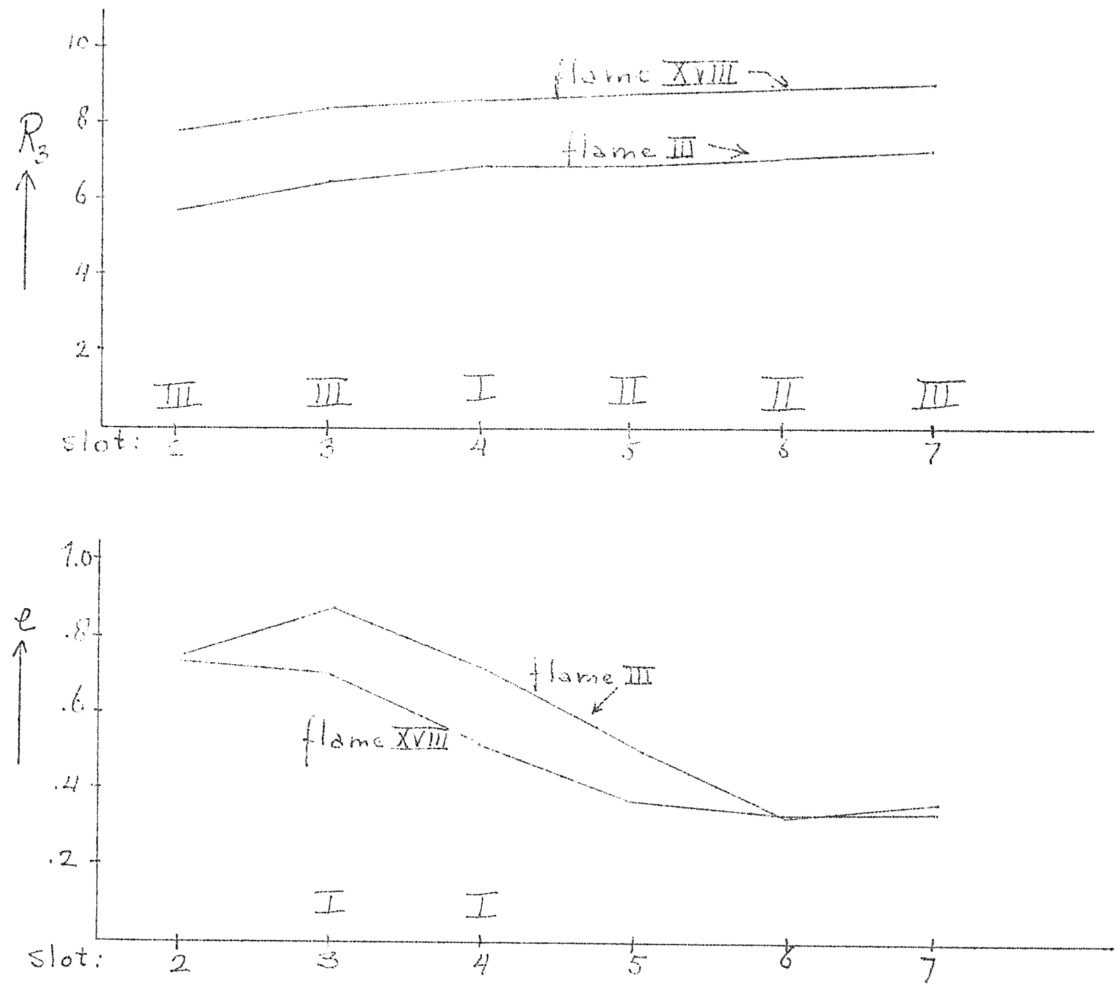
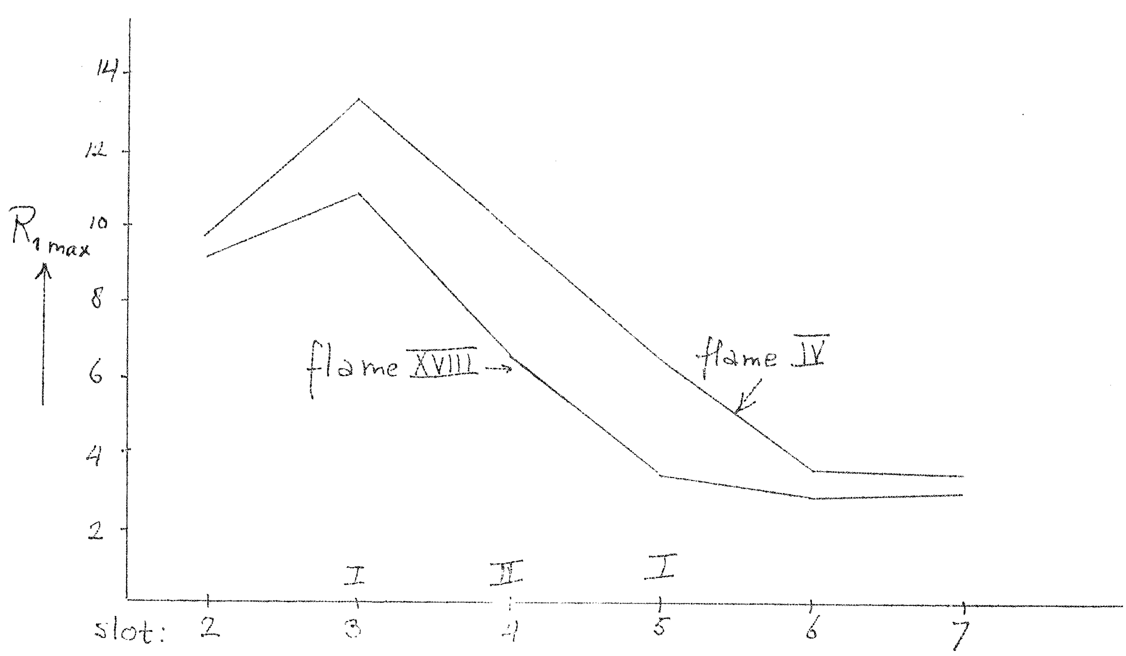
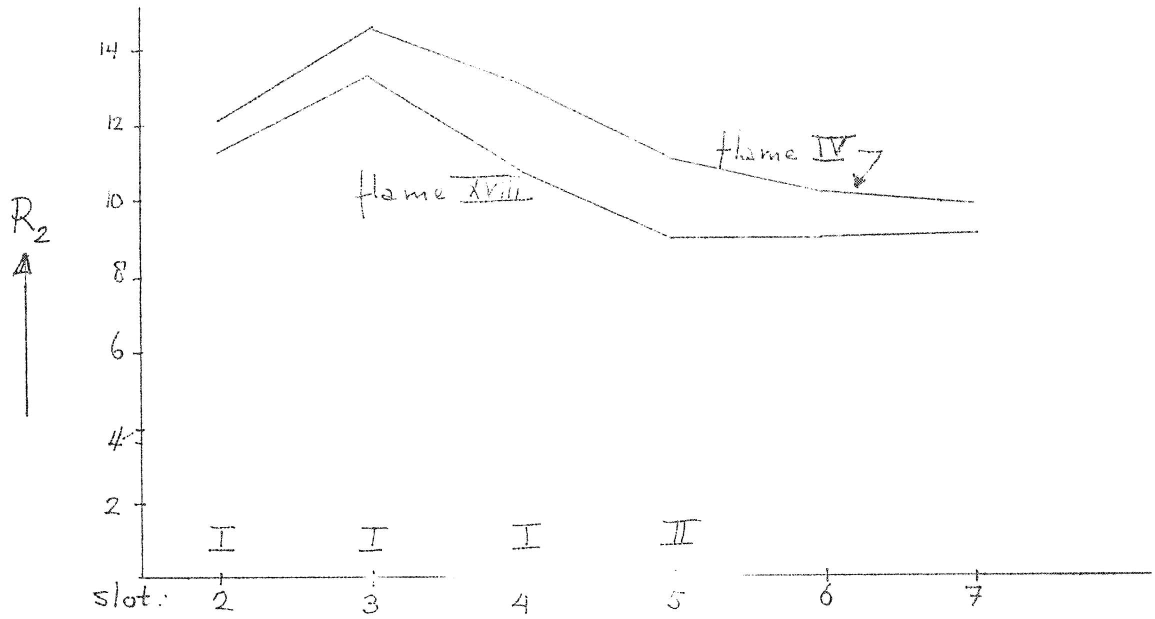
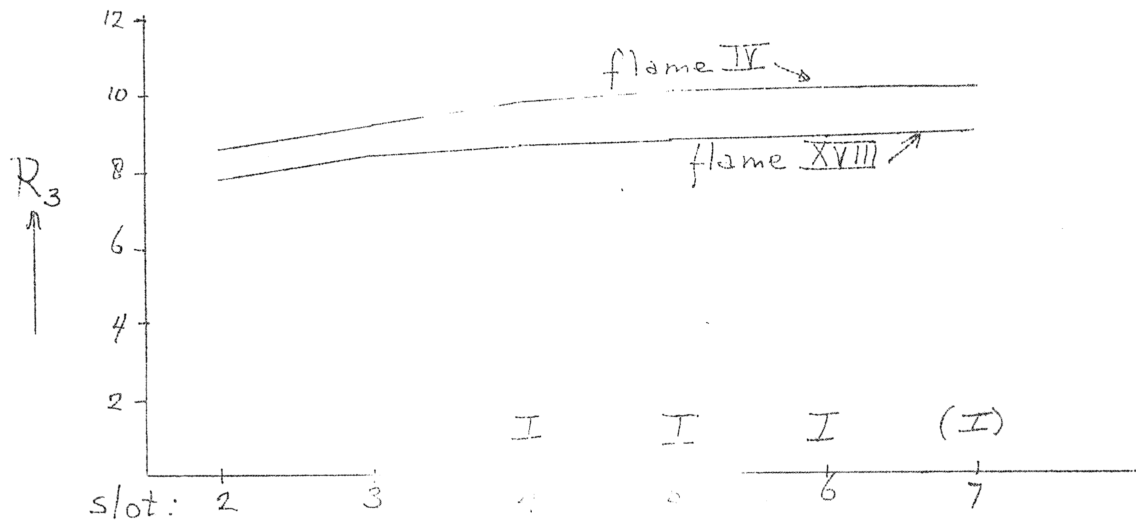
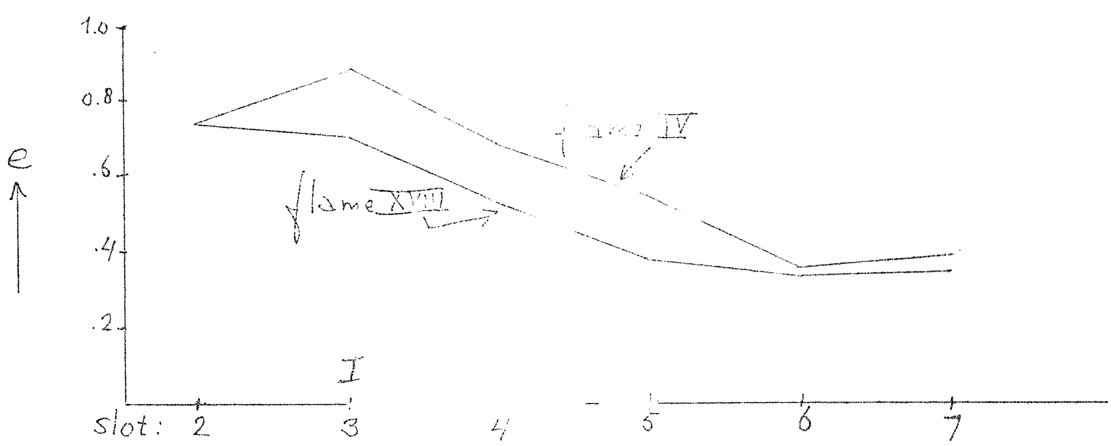


Fig. 6.4: Comparison of the flames XVIII and IV:




R_2
14
12
10
8
6
4
2
flame IV
flame XVIII
I
I
I
II
slot: 2
3
4
5
6
7


R_3
12
10
8
6
4
2
flame IV
flame XVIII
I
I
I
(I)
slot: 2
3
6
7


e
1.0
0.8
.6
.4
.2
flame IV
flame XVIII
I
slot: 2
3
4
5
6
7